# **MASTER ICFP 2019 – 2020**

# GENERAL RELATIVITY

# **SUMMARY**

This course aims at presenting a selection of advanced topics in gravitational classical dynamics. It starts with a reminder of the basics of general relativity and Riemannian geometry. The equations of motion are displayed, and Schwarzschild and Kerr solutions obtained. The black-hole properties of the latter are then discussed with special emphasis on their thermodynamic interpretation. Follows the general study of the energy-momentum tensor accompanied with the analysis of the conserved charges of the gravitational field. The last part of the course is devoted to Cartan's formalism and exterior calculus. These are the building blocks for cosmological applications and primer tools for gravitational instantons, useful for the quantum theory.

## PREREQUISITE KNOWLEDGE

Special relativity and basics of general relativity: elements of differential geometry, Schwarzschild solution, tests of Einstein's theory of gravity. This can be found in any modern textbook – see bibliography.

#### **PROGRAMME**

- 1. Thu Sept 5 course 1 From Newton to Einstein along geometry, Lie derivative, covariant derivative
- 2. Wed Sept 11 course 2 Torsion, Riemann, Weyl, geodesics
- 3. Wed Sept 18 tutorial 1 *Killing vectors*
- 4. Wed Sept 25 tutorial 2 *Geodesics*
- 5. Wed Oct 2 course 3 Einstein's field equations, Killings, non-coordinate frames, Næther's theorem
- 6. Wed Oct 9 course 4 Schwarzschild solution, black-hole structure
- 7. Wed Oct 16 tutorial 3 *Binary systems*
- 8. Wed Oct 23 course 5 General black-hole properties, Kerr solution, thermodynamics
- 9. Wed Nov 6 course 6 Back to the energy-momentum tensor: conserved quantities, Komar charges
- 10. Wed Nov 13 tutorial 4 3+1 formalism
- 11. Wed Nov 20 course 7 Exterior calculus and Cartan formalism
- 12. NOT CONFIRMED course 8 Exotic items: gravitational self-duality, Bianchi foliations, instantons –
- *13.* Wed Dec 4 tutorial 5 *Cartan formalism and GHY boundary term*
- 14. Wed Dec 18 exam (written)

All lectures EXCEPT FOR THE FIRST ONE are scheduled at the ENS, rue Lhomond, room L361 from 9:00 to  $12:30 \pm 15$ .

For the first lecture the time is 14:00 to 18:00, room L361.

A homework is also scheduled around November.

### **KEY-WORDS**

Géométrie différentielle, gravitation, trous noirs – Differential geometry, gravitation, black holes

## **BIBLIOGRAPHY**

- 1. Modern textbooks in French:
  - 1. David Langlois, *Relativité génerale*, Vuibert level M1.
  - 2. Nathalie Deruelle et Jean-Philippe Uzan, *Théories de la Relativité*, Belin level M2.
- 2. Modern textbooks in English:
  - 1. Sean M. Carroll, *An Introduction to General Relativity Spacetime and Geometry*, Addison Wesley.
  - 2. James B. Hartle, *Gravity An Introduction to Einstein's General Relativity*, Addison Wesley.
  - 3. Antony Zee, Einstein Gravity in a Nutshell, Princeton University Press.

#### 3. Classics:

- 1. Lev Landau et Evguéni Lifchitz, *Physique Théorique vol.* 2 *Théorie des champs*, MIR.<sup>1</sup>
- 2. Steven Weinberg, *Gravitation and cosmology Principles and Applications of the General Theory of Relativity*, Wiley.
- 3. Robert M. Wald, General Relativity, The University of Chicago Press.
- 4. Charles W. Misner, Kip S. Thorne, John Archibald Wheeler, *Gravitation*, W.H. Freeman and Co.
- 4. Textbooks in geometry:
  - 1. Mikio Nakahara, *Geometry*, *Topology and Physics*, Graduate student series in Physics.
  - 2. Boris Doubrovine, Anatoli Fomenko et Sergeï Novikov, *Géométrie contemporaine méthodes et applications*, MIR.<sup>2</sup>
- 5. Notes available on the web:

Matthias Blau <a href="http://www.blau.itp.unibe.ch/Lecturenotes.html">http://www.blau.itp.unibe.ch/Lecturenotes.html</a>

# **CONTACT**

 $<sup>^1</sup>$ Available also in English – Pergamon Press.

<sup>&</sup>lt;sup>2</sup>Available also in English – Springer.

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